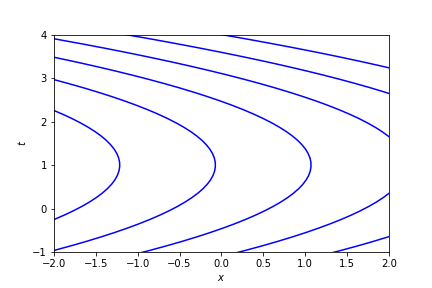
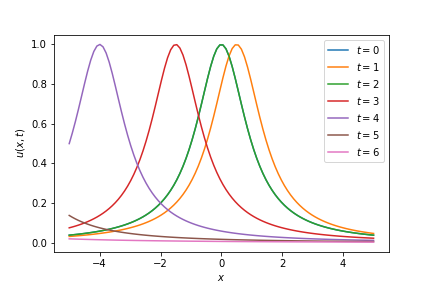
# Mathematical Methods in Engineering and Applied Science Problem Set 10.

1. Solve the initial value problem for the advection equation

Plot the characteristic curves as well as the solution u(x,t) at several different times.

For initial:

Characteristics:   
  
Solutions:



therefore figure under

1. Use the method of characteristics to solve the initial-boundary value problem:

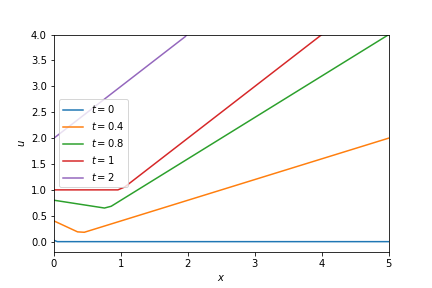
Plot the characteristic curves as well as the solution u(x,t) at several different times.

initial: ;

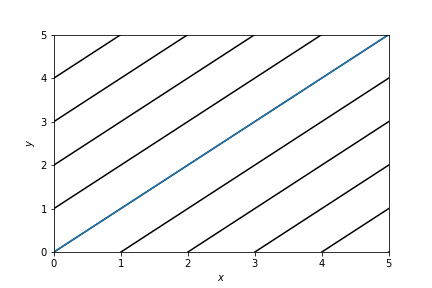
boundary: ;

Summing up:

Solution:



Characteristics:



1. Solve the initial value problem for the Hopf equation:

This equation means that u is speed.

From the initial cond. we have shock at , and fan at .

Shock speed:

Position of shock at time =0 is 1, therefore

The fan function: it intersects shock when: =>

For fan:

For shock: ( - right side).

Solution is

After this shock speed is

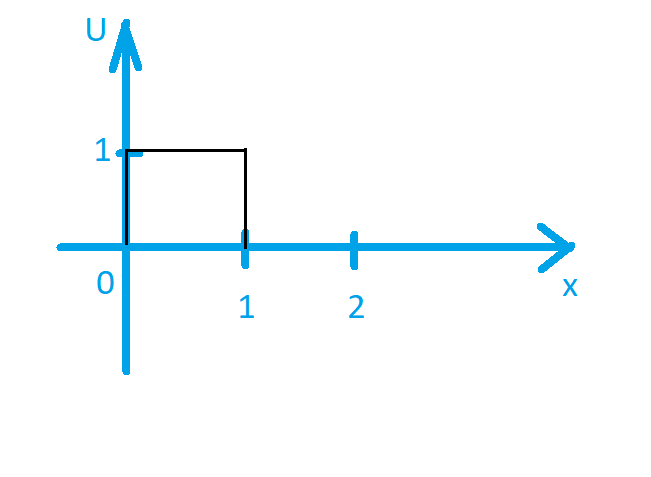
But when =>

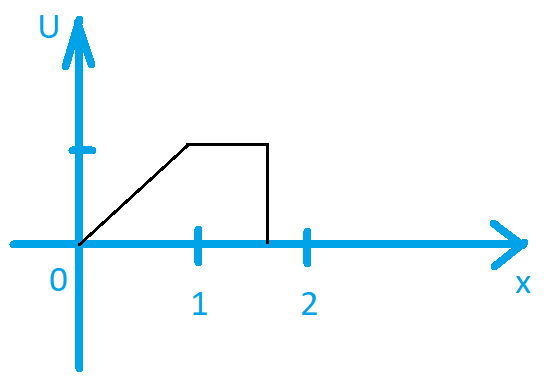
Summing up:  
: if – before fan, ) and - after shock.

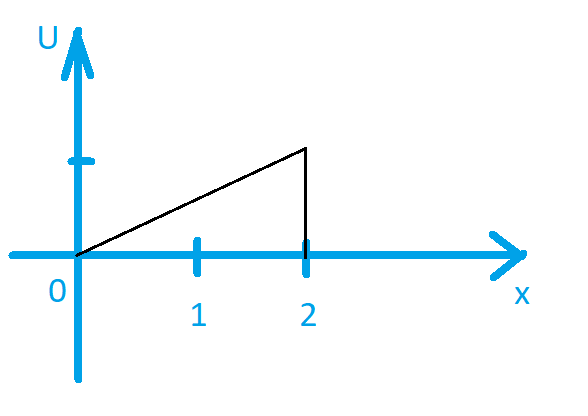
only on the start:

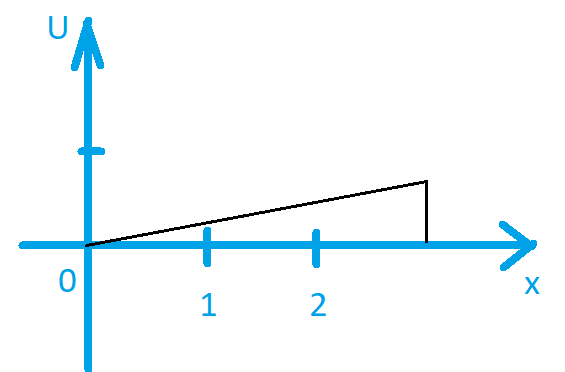
-otherwise

Let’s Plot:









1. Determine the traveling wave solutions , of the problem

with .

Let’s

as far asthen:

since

since

Since then:

Finally:

Where

1. Consider the reaction-diffusion system

with no-flux boundary conditions.

1. Analyze first the spatially homogeneous case with D = 0.

Fixed point: (0,0)

Nullclines:

1. Determine the growth rate σ of the normal modes,

Lecture formula:, where ,

Find roots:

1. For a given D, which modes are unstable? Discuss the behavior at large D and at small D.

Unstable modes if

Consider: which is always > 0 =>

Modes unstable if and .

For large D all modes are stable(except ), because

For small D number of unstable modes more, because

1. What is the largest value of D such that spatially non-uniform perturbations grow with time?

spatially non-uniform perturbations grow with time if

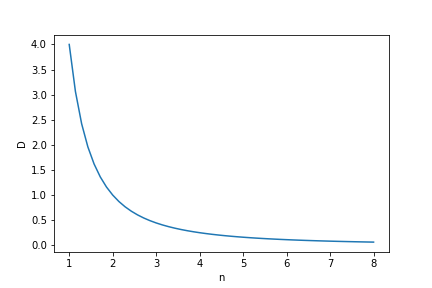
1. Plot the neutral curve, i.e. D (n) dependence for zero growth rate, Re(σ) = 0, and

indicate the regions in the D − n plane where the solution is stable and where it is

unstable

As we found before

Let’s plot



Under line zone of instability

Above line zone of stability.

1. (Extra credit). Equation describes variable-speed advection in a

certain non-uniform medium. Explain how to solve it by the method of characteristics

for general and initial data Next, specialize to

with small parameter and find an explicit form of the solution including terms

up to . Let and plot the solution at at several different or as a

surface in the -plane. What happens if ?

=>initial:

solve this with respect to , and we get